

Citation for published version:

Frenken, K, Izquierdo, L & Zeppini, P 2012, 'Branching innovation, recombinant innovation, and endogenous technological transitions', *Environmental Innovation and Societal Transitions*, vol. 4, pp. 25–35.
<https://doi.org/10.1016/j.eist.2012.06.001>

DOI:

[10.1016/j.eist.2012.06.001](https://doi.org/10.1016/j.eist.2012.06.001)

Publication date:

2012

Document Version

Peer reviewed version

[Link to publication](#)

University of Bath

Alternative formats

If you require this document in an alternative format, please contact:
openaccess@bath.ac.uk

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.



Eindhoven Centre for Innovation Studies

Innovation diffusion in networks: the microeconomics of percolation

Paolo Zeppini, Koen Frenken and Luis R. Izquierdo

Working Paper 13.02

Eindhoven Centre for Innovation Studies (ECIS),
School of Innovation Sciences,
Eindhoven University of Technology, The Netherlands

Innovation diffusion in networks: the microeconomics of percolation

Paolo Zeppini^{a,c*} Koen Frenken^a Luis R. Izquierdo^b

^a *School of Innovation Sciences, Eindhoven University of Technology*

^b *University of Burgos*

^c *CeNDEF, Faculty of Economics and Business, University of Amsterdam*

Key words: critical transition, demand, learning curves, market efficiency, social networks.

JEL classification: C63; D42; O33.

Abstract

We implement a diffusion model for an innovative product in a market with a structure of social relationships. Diffusion is described with a percolation approach in the price space. Percolation shows a phase transition from a *diffusion* to a *no-diffusion* regime. This has strong implications for market demand and pricing. We study the effect of network structure on market diffusion efficiency by considering a number of cases, such as one-dimensional and two-dimensional lattices, small worlds, Poisson networks and Scale-free networks. We consider two measures of diffusion efficiency: the size of diffusion and the diffusion time-length. We find that network connectivity “spreading” is the most important factor for the size of diffusion. Clustering is ineffective. This means that societies with higher dimensionality are better markets for diffusion. This result is most evident for the size of diffusion, while a short average path-length is more important for the speed of diffusion. Endogenous learning curves shift the percolation threshold to higher prices, and constitute an endogenous mechanism of price discrimination. The best market strategy of innovation diffusion is to start with high price and allow for a learning curve.¹

1 Introduction

1.1 Modelling innovation diffusion as percolation

The success of an innovative technology and the market penetration of a new product largely depend on the diffusion process. Seminal works on innovation diffusion (Griliches,

*Corresponding author. Email: p.zeppini@tue.nl

¹We are grateful for feedback from presentations at the College of Management, EPFL Lausanne, the LATSIS Symposium, ETH Zurich, the ABMCTS conference Agent-based models and complex technological systems, ETH Zurich, and the Economic Colloquia of the University of Amsterdam.

1957; Mansfield, 1961; Bass, 1969; Davies, 1979) have identified a number of factors that drive and sometimes delay the establishment of new technologies, as consumer heterogeneity, imperfect information, sunk costs of adoption. Yet, the role of market network structure in the diffusion of innovations is a rather recent research topic (Banerjee et al., 2012), and can play a fundamental role in the success or failure of innovations. This is particularly relevant to the problem of more efficient innovations that do not diffuse in face of less desirable incumbent technologies.

In this article we study innovation diffusion in networks with a percolation model. Percolation refers to the slow flow of liquid through a porous medium, and is a natural model to describe phenomena that present a sharp separation between a diffusion and a no-diffusion regimes (phases) (Stauffer and Aharony, 1994). Percolation models has been successfully applied to epidemiology (Davis et al., 2008). We claim that percolation is a very ‘economic’ model in that it combines the contagion mechanism of information diffusion (Bass, 1969) with the economic approach of rank models (Stoneman, 2002). Several empirical studies document the presence of these two factors behind diffusion patterns (Canepa and Stoneman, 2004). Moreover, a percolation model introduces a network structure, which allows to study local effects in adoption decisions and to derive diffusion patterns with a bottom-up approach. This gives a micro-founded explanation of S-shaped adoption time series, and allows to evaluate the effect of different market network structures on innovation diffusion.

In our model consumers are the nodes in a network of social relationships, and are heterogeneous in terms of their preferences, which are expressed as a reservation price. A consumer adopts the innovation only if two conditions are met: first, she is informed about its existence, second, the innovation price is below her reservation price. We make the two following assumptions: first, information is local, meaning that one consumers knows about the innovative product only if a neighbour consumer adopts. Second, reservation price levels are random, and follow a uniform distribution. The uniform distribution of reservation prices would lead to a linear demand in case of perfect information. We show how the percolation mechanism of innovation diffusion modifies the demand with respect to this benchmark.

The assumptions above define a social “percolation” model, where the price plays the role of the density in a material layer: the higher the price, the less likely percolation occur. Drawing consumers’ reservation prices is like randomly “shutting down” or removing nodes from the network. Consumers with too low reservation price stop diffusion

locally. Diffusion has a sizable extent only if a giant connected component remains after the removal. As it happens, the size of the connected component depends non-linearly on the innovation price, with a critical transition that separates the diffusion from the non-diffusion regime.

A model of social diffusion suggests a number of variants. We consider two major modifications of the traditional percolation model. First, we consider a number of different network structures, other than the two-dimensional lattice. Second, we introduce learning curves, with an innovation price decreasing in the number of adopters. Learning curves are a stylized fact of technological change (Wright, 1936; Abernathy and Wayne, 1974; Argote and Epple, 1990), which can strongly affect the diffusion process. With our model we address the following questions. How the market network structure affects diffusion efficiency, and what are the main structural drivers of diffusion? What are the implications of the diffusion phase transition for market demand? How endogenous learning curves affect the percolation phase transition, and for which network structures learning is most effective?

Our methodology is based on batch simulation experiments. The model is run a number of times in each parameter setting, and the parameter space is searched systematically for all different network structures. The main findings are the following. The average degree of the network has a moderate effect on the extent of diffusion, while the average path-length is far more important. Even more important is that a market presents connectivity “spreading”, meaning that neighbours of successive orders (friends of friends) are increasing in number. Connectivity spreads in networks where nodes can be arranged in more dimensions. In a society, dimensions are identified with the number of different social domains or environments, from family ties and friendship relations to professional activities, sports, leisure, and so on. The message from our analysis is that societies with higher dimensionality are better markets for diffusion.

The accent on connectivity spreading reconciles results on regular lattices with results on random networks. A popular hypothesis is that connectivity dispersion favours diffusion (Vega-Redondo, 2007; Goyal, 2007). This does not apply to regular lattices, where the degree is fixed. However, in the limit of infinitely many dimensions a regular lattice is a Poisson random network (Albert and Barabasi, 2002). By evaluating networks in terms of connectivity spreading allows to compare regular lattices and random networks.

A further result of our analysis is that markets structured as small-worlds are relatively inefficient in a percolation model, showing a relatively small diffusion size. This result

is relevant to the question whether clustering (which is relatively high in a small-world) is good or bad for diffusion: as far as diffusion is driven by a percolation mechanism, clustering is ineffective. The intuition is that clusters have many redundant links. A triplet is useless for the percolation process, since a consumer does not need to have more than a friend adopting to be informed, and the adoption decision is independent on the number of neighbours adopting.

The result on the effect of clustering is in line with empirical evidence on technology diffusion (Fogli and Veldkamp, 2012) and against experimental evidence on behaviour diffusion (Centola, 2010). For technology diffusion highly clustered collectivist societies present lower innovation diffusion rates, compared to individualist societies. On the other hand, high clustering favours behaviour diffusion, due to social reinforcement. Our percolation model offers a clear benchmark for the adoption mechanism: when innovation adoption is driven by individual preferences only, and links only bring information, clustering does not favour diffusion. This is true also in presence of local reinforcement: while small-worlds benefit strongly from learning, one-dimensional lattices do not, which indicates that learning effects are not favoured by clustering, but by a relatively low average path-length.

Learning curves have a strong impact on diffusion efficiency, shifting the percolation threshold so as to enlarge the diffusion phase. Learning is particularly effective in a two-dimensional lattice and in small-worlds. By shifting the percolation threshold, learning reduces the elasticity of demand of a structured market. This fact has strong implications for pricing in a monopoly. Learning curves can be used as a price discrimination mechanism. As with durable goods (Tirole, 1988), the best strategy is to start with a high price of the innovation, and let learning lower the price adaptively, as diffusion goes through. This allows to selectively reach the consumers with the higher reservation prices at the right time.

1.2 Models of economic diffusion

Most economic literature on the diffusion of ideas or products is based on the idea of contagion. Geroski (2000) is a survey of models that use different mechanisms to explain the stylized fact of S-shaped diffusion curves. A stronger accent on social interactions is in Young (2009), who classifies diffusion mechanism as either *social contagion*, *social influence* and *social learning*. These models deliberately discard the role of a network topology of social interactions in the diffusion dynamics.

Standard contagion models in theoretical epidemiology are the *SI* (Susceptible-Infected) model, the *SIS* (Susceptible-Infected-Susceptible) model and the *SIR* (Susceptible-Infected-Recovered) model. A review of these models is in Vega-Redondo (2007). Network models of information cascades such as Watts (2002) are slightly different from percolation, since they are generally based on social pressure as decision mechanism. Pastor-Satorras and Vespignani (2001) study epidemic spreading in scale-free networks using the *SIS* diffusion rule. Lopez-Pintado (2008) extends this model to other diffusion rules, as imitation and threshold rule. Both these models rely on a mean-field approximation of the social interaction network that allows to derive analytical results

In the game-theoretic literature models of local strategic interaction address the issue of diffusion of strategies or more generally of behaviours. Blume (1995) considers regular lattices of strategic local interactions. Morris (2000) develops analytical techniques for the study of general local interaction networks. Although the diffusion of strategic behaviours shares many aspect with the diffusion of new products or ideas, peculiar issues of strategic interactions, as revision of strategies, and best-response equilibrium constitute important differences.

In the management literature there are models that consider the role of complex network structures on product diffusion. Lee et al. (2006) address market competition of two products, and show that in some cases the network structure allows a laggard product to survive even in presence of positive adoption externalities. Choi et al. (2010) instead focus on the effect of network structure on diffusion rates and market penetration. Pegoretti et al. (2012) extend the two models above by considering competition of more than two products. Their main focus is on the role of information. They show that in a small-world network imperfect information increases the probability of one innovation becoming dominant in the market, while at the same time it favours market adoption. This is possible due to frictions between non-compatible products, which have a lower impact when there is one dominant product.

1.3 Percolation models of economic diffusion

The first percolation model of innovation diffusion has been proposed by Solomon et al. (2000). They address the metaphorical example of movies markets, and show how “hits” and “flops” can be explained by the critical transition of percolation. Moreover, whenever movies producers and consumers “adapt” by adjusting the product quality and the subjective quality requirement, respectively, the market presents a self-organized criti-

cality that drives it to the percolation threshold. Silverberg and Verspagen (2005) bring percolation into the realm of technological change, introducing a complex space where one dimension is the product performance and the other is a measure of technological distance. Their main focus is on stylized facts of technological change such as cumulativeness and clustering in time. Hohnisch et al. (2008) show how learning curves explain the empirical evidence of delayed diffusion take-off.

Frenken et al. (2008) extend the percolation model to the competition of different products in a Hotelling space. Consumers can choose to buy a product that is similar but not identical to the one purchased by the neighbour. An application of the percolation model of product diffusion to environmental economics is proposed by Cantono and Silverberg (2009), who study the effect of subsidies for green energy technologies on the percolation threshold. Iribarren and Moro (2011) is a study of information diffusion on real social networks. They show the importance of “affinity” between the carrier of the information message (the agent) and the message content in sustaining a path of diffusion. In a recent work Campbell (2012) analyses the micro-economic effects of percolation, focusing on monopoly pricing and advertising strategies.

Percolation models and epidemic diffusion network models are very much related: the *susceptible* and *infected* states of an *SI* model correspond to accessible and inaccessible states of percolation. Percolation is a peculiar model of diffusion in a network. When nodes’ accessibility is drawn from a probability distribution inaccessible nodes are “switched-off”, their links deleted, and a random network results, that we name the *operational network*. This is defined by two factors: the initial network (which can be a regular lattice or a random network) and the preference distribution. An interesting technical question is how the starting network and the preferences distribution jointly determine the resulting operational network.

Random networks present a second order critical transition in the connectivity space: there is a threshold value of connectivity such that below all connected components have negligible size, while above a component of macroscopic size is present, the so-called *giant component*. The critical transition of a giant component is at the core of all critical transitions of processes on networks, including percolation. Percolation occurs whenever a *giant operational component* is created after the draw of nodes accessibility (material porosity, consumers reservation price, etc.). In a social percolation model, the final number of adopters is the size of the giant operational component, as long as at least an initial adopter (seed) belongs to it.

Two important points must be kept in mind when studying diffusion with a percolation model. The first is that whenever we use a regular lattice, connectivity in the resulting operational network is bounded by the connectivity of the starting network. For instance, if we start with a regular squared lattice, four is the upper bound of connectivity in the operational network. The second difference hinges on the different nature of the two factors giving place to the operational network in percolation: the starting network represents physical or social connections between nodes, while the accessibility random variable represents a specific state of the node, which may change as it happens with endogenous learning curves in our model.

The reminder of this article is organized as follows. Section 2 presents the standard social percolation model, and addresses the implications for demand theory. Section 3 compares different network structures for the market. Section 4 addresses the effects on endogenous learning curves on percolation for the different networks. Section 5 concludes.

2 Percolation, diffusion and market demand

We consider the market for an innovative product, where the producer acts as a monopolist and N consumers form a network of social relationships. Consumers i and j are friends if there is a link $\eta_{i,j}$ connecting them. Such links are either existing ($\eta_{i,j} = 1$) or absent ($\eta_{i,j} = 0$), and are constant through the diffusion process. The product price assumes values in the interval $[0, 1]$, and it is assigned before diffusion starts. Consumers' preferences are expressed by a reservation price, below which the consumer adopts, and above which she does not. Reservation prices are drawn from a uniform distribution, $p_i \sim U[0, 1]$, which would define a linear demand if information was global. Drawing individual reservation prices amounts to randomly shut down nodes (Fig. 1). The in-

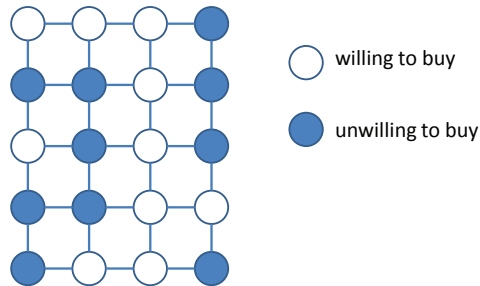


Figure 1: Drawing consumers' reservation prices amounts to randomly shut down nodes.

formation about the product is local, meaning that only the purchase by a neighbour informs an agent about its existence. If an agent does not buy the product, she does

not spread information, possibly preventing the adoption by a neighbour with a higher reservation price.

Our methodology is based on batch simulations, where we run the model a given number of times for a specified setting, and compute the average outcomes. The model is initiated by setting the initial adopters (seeds).² The percolation model is setup with the following steps:³

1. model settings: network structure, number of consumers N , product price p ;
2. draw consumer reservation prices (uniform distribution)
3. draw initial adopters (uniform distribution)

In this section we show the implications of a percolation critical transition for market demand. With this aim we consider the standard percolation framework used in physics, a regular two-dimensional lattice (henceforth the ‘grid’, Fig. 2). Different structures are studied in Section 3. The network structure of the market introduces a non-linearity in

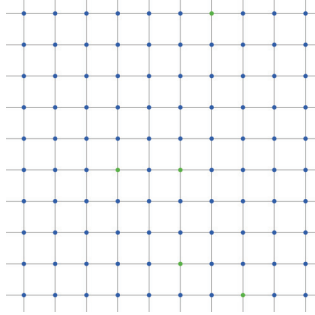


Figure 2: A grid with 100 consumers and degree 4.

the adoption process. If the innovative product has a price p , with a uniform distribution of reservation prices the probability that a consumer is *willing-to-buy* is $q = 1 - p$. If there were not local effects, the expected number of adopters would be $N(1 - p)$. Instead the social network structure of the market introduces a threshold level of the price above which product diffusion is way below its potential market penetration (Fig. 3). Below the threshold the product diffuses (percolates) and reaches its potential penetration. This is

²These can be consumers that are given the product for free. Our model also allows for a different setting where initial consumers are like other consumer, and evaluate the product based on their reservation price. We do not use this setting, because the number of seeds becomes a random variable which affects the comparative analysis of different simulations of the model in different conditions.

³The model is implemented in *NetLogo*, a programmable framework for agent-based modelling (<http://ccl.northwestern.edu/netlogo/>)

a second-order critical transition. In the non-diffusion phase, the number of adopters is negligible. In the diffusion phase, this number is almost equal to the number of willing-to-buy consumer $N(1 - p)$, and the relationships with the product price is linear as it would be without a network structure.

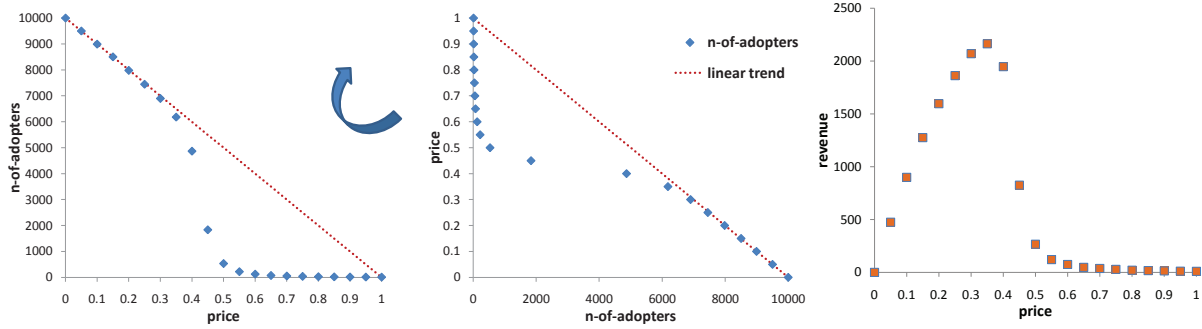


Figure 3: Simulated diffusion in a percolation model with a two-dimensional lattice. Left: number of adopters as a function of the product price. Centre: adoption values reported as a demand curve. Right: monopoly revenues. Model setting: $N = 10000$ consumers, 10 seeds. Values are averages over 50 simulation runs. The standard deviation is maximum at the critical transition (about 25%).

In Fig. 3 the values reported for the final number of adopters (squared symbols) are averages from 50 simulation runs, which are repeated in each setting of the model (here is the product price p that changes in the range $[0, 1]$ with steps $\Delta p = 0.05$). The dashed red line represents the theoretical values $N(1 - p)$, that is the expected number of adopters without local effects. In this simulation experiment there are 10000 consumers arranged in a *thorus* (a 100×100 grid wrapped horizontally and vertically, which rules out boundary problems), with ten *seeds*. The diffusion is very low above 0.5. Between 0.5 and 0.4 there is an abrupt increase of the final diffusion size, towards full efficiency below 0.4. The critical transition of percolation is not sharp due to the finiteness of the network. For an infinite two-dimensional lattice with degree 4 the percolation threshold is about 0.407.⁴ Different regular lattices have different percolation thresholds (Stauffer and Aharony, 1994). These values are a static topological characteristic of the lattice.

The negative effect of a percolation mechanism on diffusion size is a market inefficiency due to local information effects. Many consumers with high reservation price are willing to buy the innovative product above the threshold, but do not buy because they are not informed.

The total number of adopters is a measure of the aggregate market demand (Fig. 3, central panel). Without a network structure, if all consumers would be exogenously informed, we would have a linear demand expressed by $Q = N(1 - p)$. The percolation

⁴Often percolation is defined in the dual space $q = 1 - p$, where the threshold is $1 - 0.407 = 0.593$.

phase transition strongly affects the demand (Campbell, 2012). Consider the market equilibrium where demand equals supply, and assume the monopolist sets supply where the marginal revenues equal marginal costs, as in a standard monopoly framework. Above the percolation threshold, the demand is more elastic. In a monopoly market, the price corresponding to the quantity level where marginal revenues equal marginal costs is lower due to the percolation phase transition. The best strategy for a monopolist is to price just above the threshold. This carries a large increase of adoption for a relatively small increase of the price (Fig. 3), and consequently an increase of monopoly revenues, the price that maximizes revenues is just above the threshold. (Fig. 3, right panel).

3 Market network structures

Different network structures are expected to present different features of percolation critical transition. Here we consider two-dimensional regular lattice, one-dimensional regular lattice, Small-worlds as defined by Watts and Strogatz (1998), Poisson random networks, and Scale-free random networks. Moore and Newman (2000) provide analytical results for percolation thresholds in one dimensional small-world networks. Newman et al. (2002) introduce a formalism for the study of percolation in two-dimensional small-world networks. We compare these different network structures by looking at the following diffusion outcomes: the extent of diffusion, measured by the final number of adopters, the resulting demand curve, the monopoly revenues associated to such demand and the speed of diffusion, measured by the time required to reach an equilibrium value of adoption.

We consider two levels of network connectivity (number of links per node), a degree 4 and a degree 8 (for the power law network we use degree 2 and degree 4). For random networks these values are the mean of the degree distribution.

3.1 Two-dimensional lattices

We start by considering again a two-dimensional lattice with degree 4. In Fig. 4 we report the diffusion time-length (right) beside diffusion size (left). The time-length of diffusion has a pronounced spike which is typical of phase transitions (Watts, 2002). At the threshold price the percolation operational component has a macroscopic size already, but many nodes are connected with only one link, which makes diffusion slow. The time-length spike can be used to locate the critical transition threshold, which here occurs at $p = 0.4$. The threshold value gives an indication of the optimal price for a monopolist.

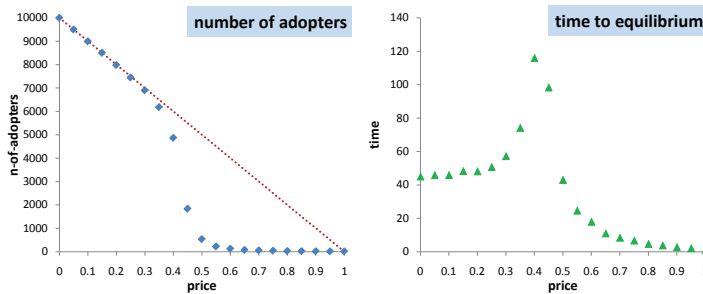


Figure 4: Percolation in a Two-dimensional lattice with degree 4. Left: final number of adopters. Right: diffusion time-length. Values are 50 runs averages. The standard deviation of the number of adopters is maximum at the threshold (about 25%). For the diffusion time-length it is larger in the no-diffusion phase (about 40%).

Moreover, above the threshold diffusion is relatively faster, which is another desirable aspect.

A higher connectivity of the market network is expected to enhance diffusion. This is what happens, as testified by simulation results reported in Fig. 5. The percolation

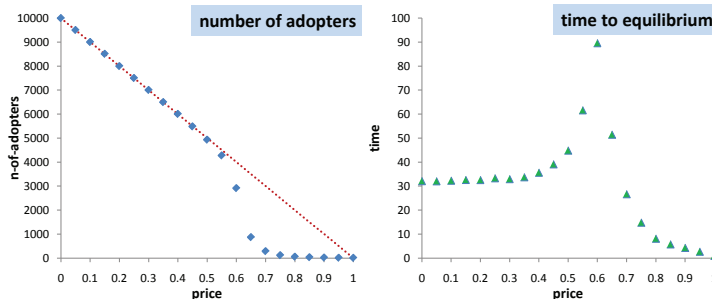


Figure 5: Percolation in a Two-dimensional lattice with degree 8. Left: final number of adopters. Right: diffusion time-length. Values are averages over 50 runs. The standard deviation for adoption size is 40% at the threshold, 25% in the no-diffusion phase, and below 1% in the diffusion phase. For diffusion time the standard deviation is 35% in the no-diffusion phase and 10% in the diffusion phase.

critical transition occurs at a threshold price $p \simeq 0.6$. A higher threshold means that local effects in a market with higher connectivity are less severe, and consequently the demand elasticity above the threshold is less affected. This allows the monopolist to charge a higher price. We may conclude that higher connectivity is good for the extent of diffusion, but bad for consumers, who are better off when they are less informed Campbell (2012).

3.2 One-dimensional lattices

A completely different way of arranging nodes is a one-dimensional lattice (“circle” in short). This can be figured placing nodes on a circle, and connecting a given number of neighbours on either side. Fig. 6 is an example with 20 nodes and degree four. One-

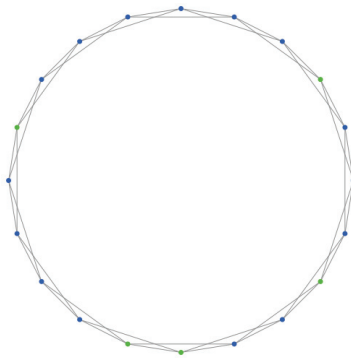


Figure 6: A circle with 20 nodes and degree 4.

dimensional lattices and two-dimensional ones are very different, even if the degree is the same. The structural difference is well expressed by the average path-length and the clustering coefficient (Wasserman and Faust, 1994). The first is a measure of how many steps it takes to cover the average distance between two any nodes, and gives an indication of how closely connected is the network. The second measures the relative number of clusters. Technically this is given by the number of triads. Loosely speaking, it tells how often two neighbour nodes are also neighbour of each other. Compared with two-dimensional lattices of the same degree, one-dimensional lattices have much larger average path-length and and clustering (the latter is zero in two-dimensional lattices with degree 4).

Fig. 7 and Fig. 8 report simulation results of the percolation model for one-dimensional networks with degree 4 and 8, respectively. In this case we run only 20 simulations for each setting, due to a longer values of diffusion time-length. One-dimensional lattices

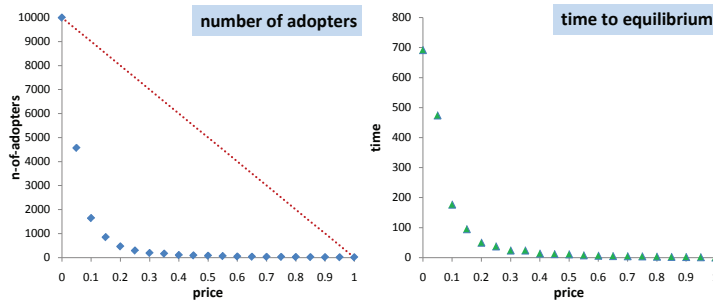


Figure 7: Percolation in a One-dimensional lattice with degree 4. Left: final number of adopters. Right: diffusion time-length. Values are averages over 20 runs. Standard deviation is 10% - 25% for the number of adopters and 20% - 40% for the diffusion time-length.

present much lower diffusion levels than two-dimensional lattices. This translates into lower prices and lower revenues for the monopolist. For a given connectivity, the one-dimensional structure is less efficient than the two-dimensional one: the grid with degree 4 does better than the circle with degree 8. This indicates that average connectivity is

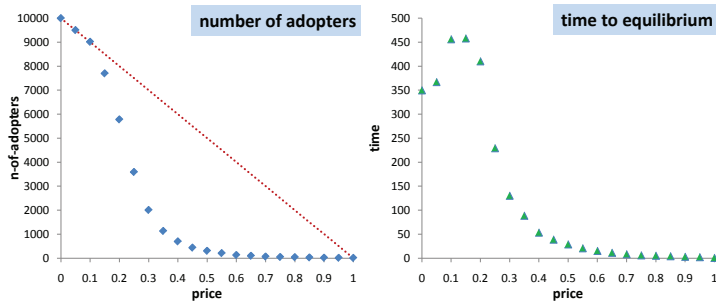


Figure 8: Percolation in a One-dimensional lattice with degree 8. Left: final number of adopters. Right: diffusion time-length. Values are averages over 20 runs. Standard deviation is 10% - 25% for the number of adopters and 25% - 40% for the diffusion time-length.

not very important for the diffusion in a network. A crucial aspect for diffusion is whether the network connectivity “spreads”. Starting from any node, one should consider how the number of successive order neighbours evolve. In a circle with degree 4 the r -order neighbours (number of neighbours after r steps) are always 4. In a grid of degree 4 the number of r -order neighbours is $4r$: with one step 4 neighbours can be reached, with two steps 8 neighbours, 12 with three steps, and so on. Topological “spreading” multiplies diffusion size.

The spreading of connectivity reflects onto the average path length. Consider the limit case of only one initial adopter. Assume that all consumers are willing to buy. In order to cover the entire market, we must look at the diameter of the network (the maximum distance between two nodes). For a regular grid of degree 4 and N nodes, the diameter is \sqrt{N} . For a circle of degree 4 and the same number of nodes, the diameter is $N/4$. If $N = 10000$, adoption in a circle must “travel” for distances that are 25 times longer than in a grid, to reach the whole network.

The low diffusion extent of circles reflects in a very high elasticity of demand. Due to a low percolation threshold, the monopoly price mark-up is very little compared to the two-dimensional lattice. As a consequence, equilibrium prices are much lower in a market structured as a circle than in a market structured as a grid.

3.3 Random networks

In this sub-section we study percolation in a number of different random networks, namely Small-worlds, Poisson networks and Scale-free networks. Small-worlds were introduced by Watts and Strogatz (1998). They are constructed starting with a regular one-dimensional lattice as the ones of Section 3.2. One defines a *rewiring* probability μ based on which any link can be re-wired. Fig. 9 shows examples with $N = 50$ nodes and degree 4 (the

total number of links is $4 \times 50/2 = 100$). In the middle panel there is a small-world network where eleven links have been rewired (the rewiring probability was $\mu = 0.1$). For

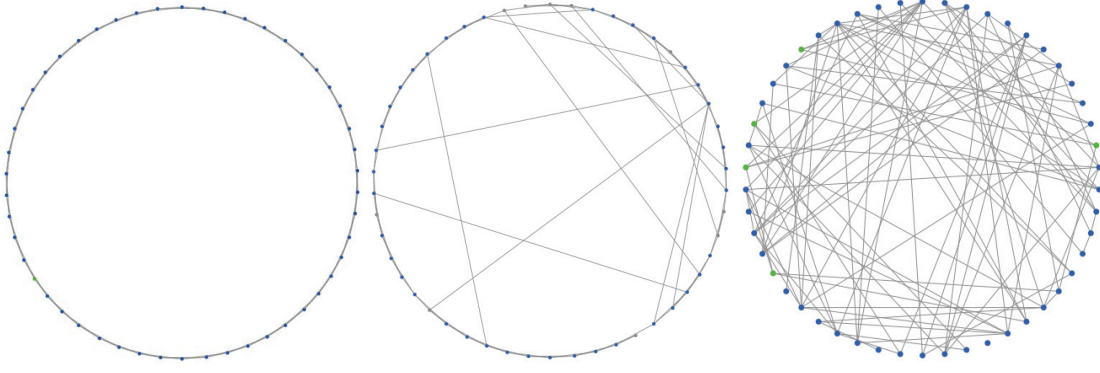


Figure 9: Small-worlds construction process: example with 50 nodes and degree 4. Left: regular lattice ($\mu = 0$). Centre: Small-world with $\mu = 0.1$. Right: Poisson network ($\mu = 1$).

$\mu = 0$ we have the starting regular lattice, while for $\mu = 1$ we have a fully random Poisson network of the type introduced by Erdos and Renyi (1960). It is important to notice that while the rewiring process strongly affects the degree distribution, the average degree remains unchanged, since the numbers of nodes and links are fixed. In the examples of Fig. 9 the average degree is 4.⁵

Figures 10 and 11 report simulation results for percolation in a small world with rewiring probability $\mu = 0.01$ and degree 4 and 8, respectively. Fig. 10 should be compared to Fig. 7 and Fig. 4. As we see, the diffusion extent is still relatively low, and

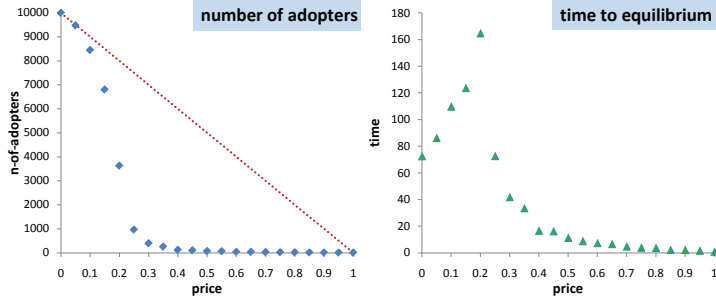


Figure 10: Percolation in a small-world (rewiring probability $\mu = 0.01$) with average degree 4. Left: final number of adopters. Right: diffusion time-length. Values are averages over 10 runs. Standard deviation for diffusion size is 40% at the threshold and lower elsewhere, and 10% - 40% for diffusion time.

little better than in the circle. This means even if the average path length is much reduced due to the rewiring process, the critical transition of percolation still occurs at relatively

⁵The degree distribution of a Poisson random network is $p(k) = \frac{1}{k!} e^{-z} z^k$, where k is the degree, and the parameter z is the average degree. Each possible link has a probability q such that, given the total number of nodes N , the average degree $z = qN$ is constant (Vega-Redondo, 2007).

low values of price. Demand is still very elastic, giving a relatively low monopoly price mark-up. Equilibrium prices increase little with respect to the regular circle. Similar considerations hold true for the case of a degree 8.

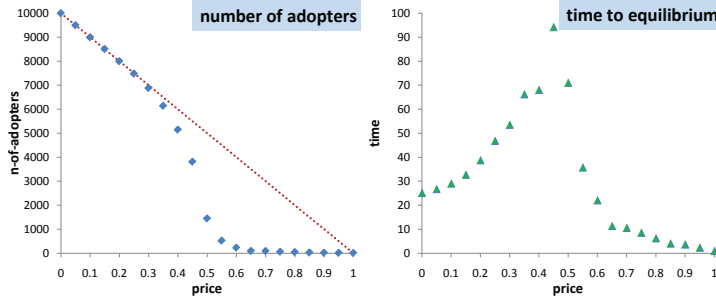


Figure 11: Percolation in a small-world (rewiring probability $\mu = 0.01$) with average degree 8. Left: final number of adopters. Right: diffusion time-length. Values are averages over 10 runs.

Small-worlds do a little better in terms of diffusion time-length. The peak value of the time required to reach an equilibrium, which is always at the threshold, is reduced about four times both for degree 4 and degree 8. This is an indication that a reduced average path-length affects more the diffusion time than the diffusion size.

Percolation in a Poisson random network (rewiring probability $\mu = 1$) are shown in Fig. 12. The critical transition occurs at a relatively high price, meaning that diffusion

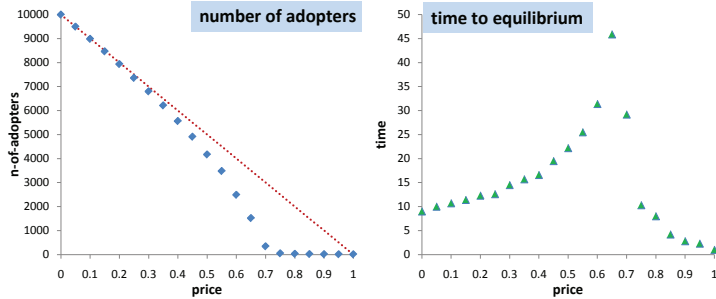


Figure 12: Percolation in a Poisson network (average degree 4). Left: final number of adopters. Right: diffusion time-length. Values are averages over 10 runs.

works very well in a fully random Poisson network. Consequently, the demand is less elastic than in Small-worlds. In a monopoly, equilibrium prices can be higher. Random networks present a smoother critical transition with respect to regular structures. This is due to the dispersion of degree across nodes, which reflects into a broad critical range of values that separate the non-percolating from the percolating phases.

Percolation results in terms of diffusion size for different networks are compared in Fig. 13 and Fig. 14 for degree 4 and degree 8, respectively. The effect of rewiring on diffusion is strong: with only 10% of links rewired, the degree 4 small-world network is closer to the

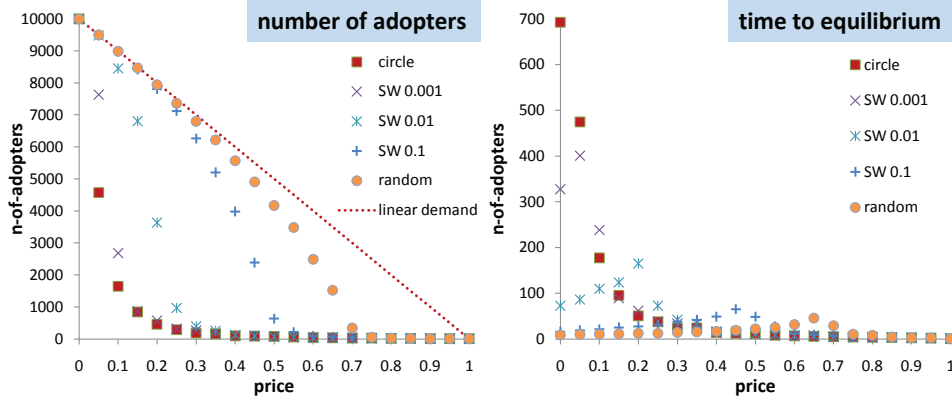


Figure 13: Percolation in different networks with degree 4. Left: final number of adopters. Right: diffusion time-length.

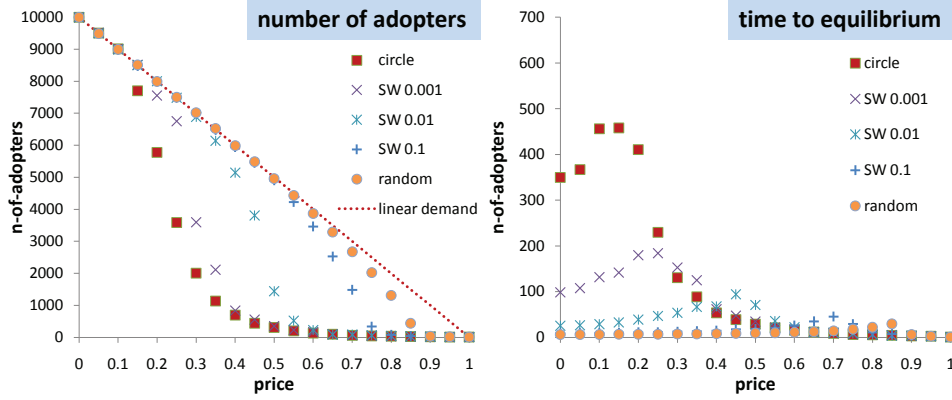


Figure 14: Percolation in different networks with degree 8. Left: final number of adopters. Right: diffusion time-length.

random network than to the starting circle. And with just as low as 1% of links rewired, the degree 8 small-world network is mid-way between the starting circle and the random network. Usually the diffusion results on small-worlds are ascribed to the virtue of long-distance weak ties, as the rewired links are referred to. The intuition is that rewired links strongly reduce the average path length, while leaving practically untouched the clustering coefficient. Fig. 15 reports clustering coefficient $C(p)$ and average path-length $L(p)$ in Small-World networks from Watts and Strogatz (1998). The combination of low average path-length and high clustering characterizes the typical small-world network, which in our theoretical setting is obtained with a rewiring probability $\mu = 0.01$. The popular hypothesis is that such combination is good for diffusion. Our results challenge this hypothesis. In Fig. 16 we compare the final number of adopters for a selection of networks studied so far. Two groups of networks show similar diffusion results. In the first group are the grid with degree 4, the small-world with degree 4 and $\mu = 0.1$ and the small-world with degree 8 and $\mu = 0.01$. The grid performs less well in terms

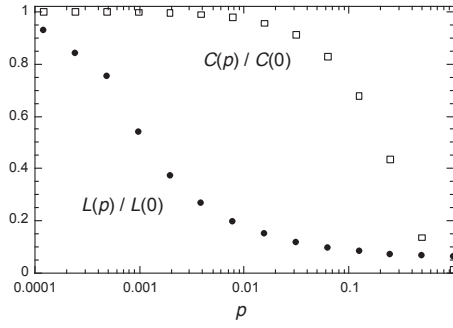


Figure 15: Clustering coefficient $C(p)$ and average path-length $L(p)$ in Small-World networks.

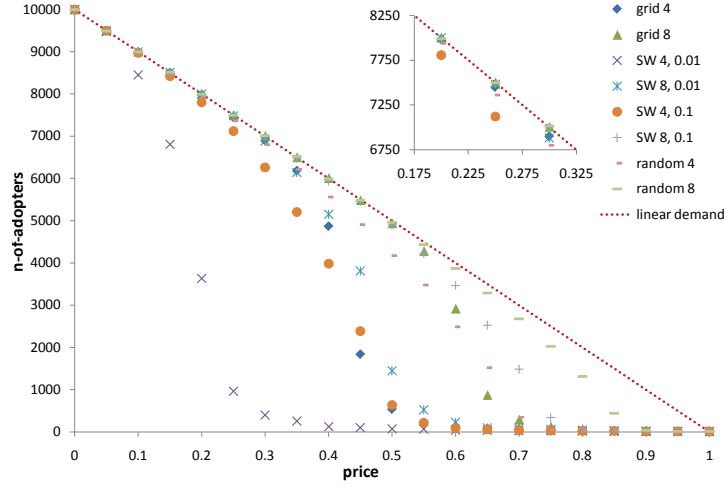


Figure 16: Percolation in different networks with degree 4 and 8.

of diffusion size below the threshold, but better above the threshold. In particular, the onset of Fig. 16 shows how the grid performs better even than the random network at low prices. The second group of networks consists of the grid with degree 8 and the Poisson network with degree 4. Again, the grid performs less well above the threshold and better below. The message from this analysis is the following: the crucial factor for diffusion as percolation is not the average path-length itself, but the topological “spreading” of the network, a structure where neighbours of successive order are increasing in number (see Section 3.2). In a random network the number z_r of neighbours at distance r is given by (Vega-Redondo, 2007)

$$z_r = \left[\frac{z_2}{z_1} \right]^{r-1}. \quad (1)$$

Connectivity spreads whenever the second order neighbours are in larger number than direct neighbours, $z_2 > z_1$. This is the case if the connectivity variance is large enough, $\langle k^2 \rangle > 2\langle k \rangle$. For Poisson random networks this simplifies to $\langle k \rangle > 1$. In other words, nodes must have more than one link, on average.

Usually a low average path-length and topological spreading are associated Albert

and Barabasi (2002). But small-world show that this is not necessarily the case: a small-world with $\mu = 0.01$ has a quite low average path-length (Fig. 15), but presents diffusion sizes that are by far lower than a random network or even a grid with same degree. Whenever diffusion is based on a percolation mechanism, a small-world only works well when it gets close to a random network. This means that connectivity spreading is the key-factor, which is high in random networks and in grids. Clustering is quite high in a small-world with $\mu = 0.01$, but this does not help diffusion quite at all. Clustering is ineffective in a percolation process, not entering either the information diffusion and the adoption decision. This result is in line with empirical evidence on technology diffusion Fogli and Veldkamp (2012), and against results on behaviour diffusion in online social networks experiments Centola (2010). This suggests that a percolation model may be better suited for technological innovation diffusion than for behaviour diffusion.

As a final remark, we observe that random network often do not present full diffusion at zero price, and regular lattices have higher diffusion sizes at low prices. The reason is that a random network may present unconnected component, which are never reached by the information about the new product, so that some consumer never buy even at zero price.

Often social networks present a “hubs” structure, which is not captured by Small-worlds or Poisson random networks. Few nodes, the hubs, have many links (high degree), while the majority has only a few links (low degree). The hubs network structure is characterized by degree distribution that follows a power law, or *Pareto* distribution. Such random networks were studied extensively for the first time by Barabasi and Albert (1999). They noticed an important characteristics of networks with hubs, which is a “scale-free” distribution. Technically such distribution is linear on a double logarithmic plot. The implication is that at any scale or value of the degree, the probability of occurrence of nodes with such degree “scales” with the same rate. Scale-free networks are found in many socio-economic systems (Albert and Barabasi, 2002). Here we study how percolation works on scale-free networks. Fig. 17 compares diffusion sizes in two scale-free networks with a small world and a Poisson random network. We consider scale-free networks with average degree 2 and 4. The case of degree 4 shows that scale-free networks are quite efficient in terms of diffusion size, being comparable with the Poisson random network. In particular, the scale free-network with degree 4 has a critical transition threshold at a higher price than the random network, meaning that it favours diffusion when the price is relatively high. Below the threshold the Poisson network catches-up

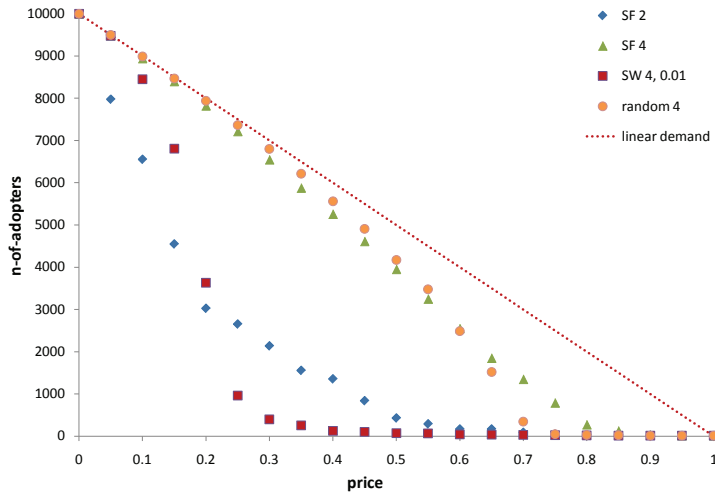


Figure 17: Percolation in scale-free, small-world and Poisson networks.

and overcomes the scale free-network. The reason is that when the innovation price is relatively high hubs are useful, because whenever a hub adopts the innovation, it can “test” many neighbours and very likely find some with reservation price high enough. This effect is even more striking when comparing a scale-free network of average degree 2 to a small-world of average degree 4: below $p = 0.6$ percolation in the scale-free network takes off, with sizeable diffusion values where the small-world networks have negligible results. The critical transition in small-worlds is sharper though, and for low prices diffusion is larger than in the scale-free network. These observations point to a trade-off in the effect of hubs compared to the small-world structure: hubs favour diffusion when this is more difficult (high prices), but fails to reach high diffusion sizes when it is easier. In that regime, a low average path-length is more effective. One final remark: if we look at the diffusion results in terms of a demand curve, the scale-free network with degree 2 presents a relatively constant elasticity, which is the shape of a hyperbolic demand.

The number of initial adopters is important whenever we deal with diffusion in a finite network. In a physical percolation model the usual assumption is a layer with an infinite horizontal dimension, and an infinite number of seeds (the liquid) located on one edge. In other words, only the finite dimension counts, and percolation occurs whenever the liquid goes through the layer. This is by no means a good schematization of a socio-economic system. We have run a number of batch simulations to study which network structures are more sensitive to the number of seeds. In Fig. 18 we report percolation results for five different network structures with 10 seeds and 100 seeds. Obviously a larger number of seeds positively affects the diffusion size. This effect is more pronounced in less efficient networks, as the circle and the small-world. In Poisson and scale-free networks the effect

is negligible. Notice that in all networks when the price is high and above the percolation threshold the diffusion time-length is shorter with 10 seeds than with 100 seeds. By no means more seeds slow down diffusion, but when seeds are too few the diffusion size is small and the process ends relatively soon.

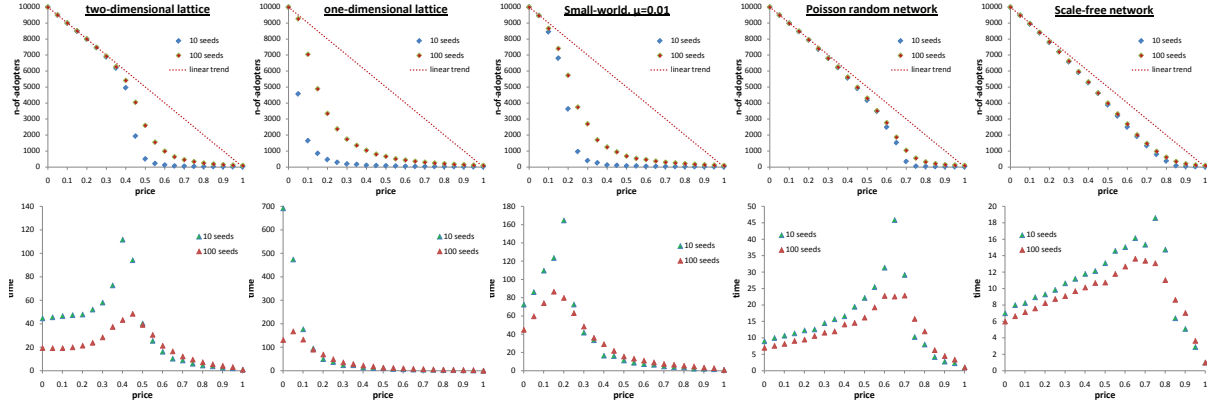


Figure 18: Effect of the number of seeds on different networks (average degree 4).

4 The effect of technological progress

The price of an innovative technology or product may change during diffusion. For technological innovations there is wide empirical evidence of so-called *learning curves*, a stylized fact showing that prices fall down as output quantity increases (Argote and Epple, 1990). There are many reasons behind learning curves: technological progress, upfront investment costs, economies of scale, learning-by-doing. In our model a proxy for output is the number of adopters. We model endogenous learning by assuming the following relationship between the price p_t and the number of adopters N_t at time t :

$$p_t = pN_t^{-\alpha}, \quad (2)$$

where p is the initial price and α is the price reduction rate. The value of α is an empirical issue, and varies across sectors. For our purposes a value around 0.2 is a good estimate.⁶

A stylized fact of innovation diffusion is a S-shaped time series of diffusion size (Rogers, 1995). For illustrative purpose we report this time series in five different networks, without and with endogenous learning. In the five examples of Fig. 19 the innovation price is $p = 0.1$, which is well into the diffusion phase for all networks but the circle, for which we are just at the threshold. The percolation model is able to reproduce the S-shape

⁶Learning curves are difficult to measure because often price reduction goes along with increased product quality or increased profitability of the technology under study. As a consequence, falling prices are always an underestimate of technological progress.

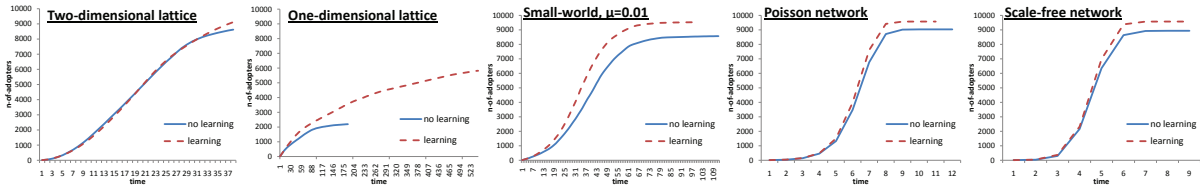


Figure 19: Adoption time series in five different networks, without learning ($\alpha = 0$) and with learning ($\alpha = 0.1$). The innovation price is $p = 0.1$ in all examples, with $N = 10000$ consumers and 10 seeds.

of empirical diffusion patterns. Different networks present different features, in terms of steepness of the curve and position of the inflection point. The features of innovation diffusion curves have been explained with learning mechanisms of adoption dynamics (Young, 2009) and with rational expectations logic combined with social interactions (Brock and Durlauf, 2010). Our illustrative comparative analysis shows how the time-pattern of diffusion dynamics is also influenced by the topological structure of social interactions.

Learning curve introduce a positive feedback in the model. Although the network topology is unaffected, the ‘accessibility’ of nodes increases with diffusion. The network does not just ‘affect’ but is also ‘affected’ by the diffusion dynamics, so that network and diffusion co-evolve. Different network structures are affected differently by endogenous learning. Fig. 20 reports percolation results from simulations on a two-dimensional regular lattice. The left panel reports the final number of adopters as a function of

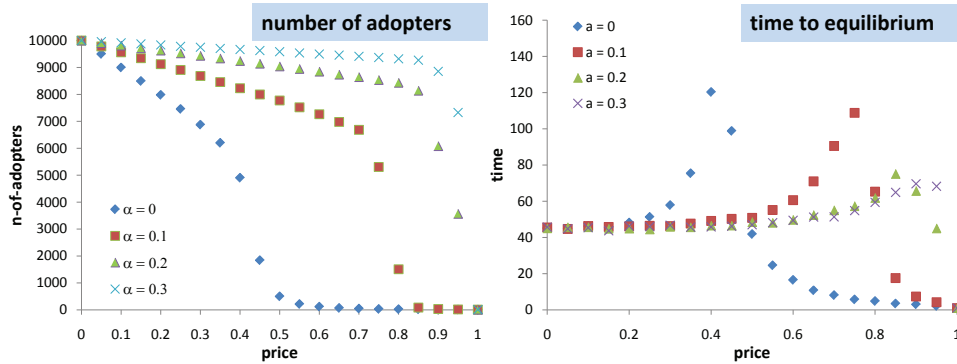


Figure 20: Percolation model with an endogenous learning curve. Two-dimensional lattice with degree 4. Left: final number of adopters. Right: diffusion time-length.

the initial price value. The first effect of learning is a shift of the percolation critical transition threshold to higher prices, as it is also testified by a shift of the diffusion time-length peak in the right panel. The diffusion phase enlarges due to learning, and one obtains sizeable market penetration for values of the initial price where it is negligible with a fixed price. The relative effect of learning is indeed larger in this price range, that in the example of Fig. 20 is $[0.5, 0.8]$. The implication for market equilibrium and pricing

are also important. Although it is difficult to visualize a demand curve that change in time due to diffusion, learning allows to charge a higher price initially, and lower it adaptively as adoption takes place. In other words, learning is an endogenous adaptive price discrimination mechanism that resembles price discrimination for durable goods (Tirole, 1988), based on which a monopolist can extract more surplus from demand.

Fig. 21 shows how learning affects percolation in a one-dimensional regular lattice. The effect is lower and more gradual than in two-dimensional lattices. In particular we notice that one-dimensional lattices present quite similar shapes of diffusion size and diffusion time as functions of price. This means that diffusion in one-dimensional networks is a linear process, so that size and time of diffusion are interchangeable. The intuition is that a one-dimensional lattice does not “spread”, and adoption has only one route to travel. The linearity of diffusion in one-dimensional lattices is also testified by the overall

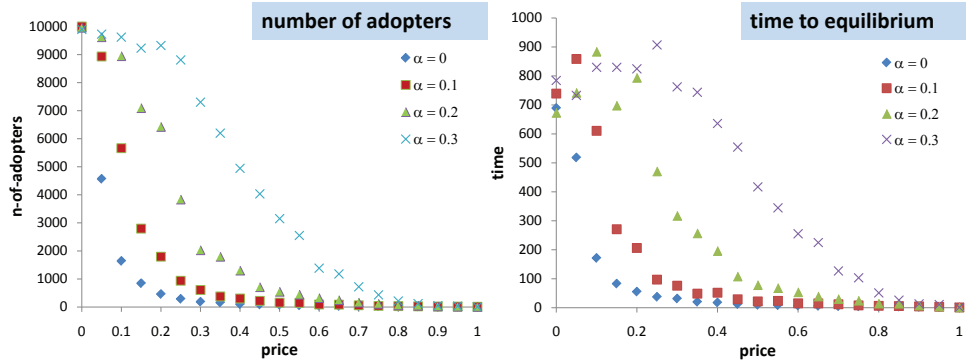


Figure 21: Percolation model with an endogenous learning curve. One-dimensional lattice with degree 4. Left: final number of adopters. Right: diffusion time-length.

increase in the diffusion time-length in the whole price range, which reflects a larger size of potential adopters to fulfill.

In a small world endogenous learning has a strong effect (Fig. 22). The relative shifts

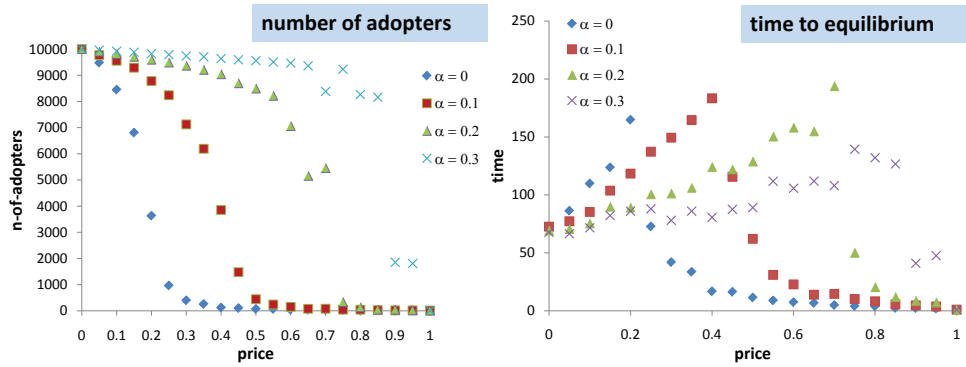


Figure 22: Percolation model with an endogenous learning curve. Small-world (rewiring probability $\mu = 0.01$) with average degree 4. Left: final number of adopters. Right: diffusion time-length.

of the percolation threshold are comparable to a two-dimensional network. A learning rate $\alpha = 0.2$ moves the percolation threshold by half the price range, from 0.2 to 0.7. An explanation resides on the fact that a small-world represents a middle state between two very different structures as the one-dimensional lattice and the Poisson network, which makes it very sensitive to changes. Learning ‘switches-on’ links. Re-wired links are very important, as they make the average path-length low. Whenever learning happens to switch-on a re-wired link, diffusion has a sudden increase, because a new region of adopters is accessed. Since re-wired links are relatively few (only 1% in the example of

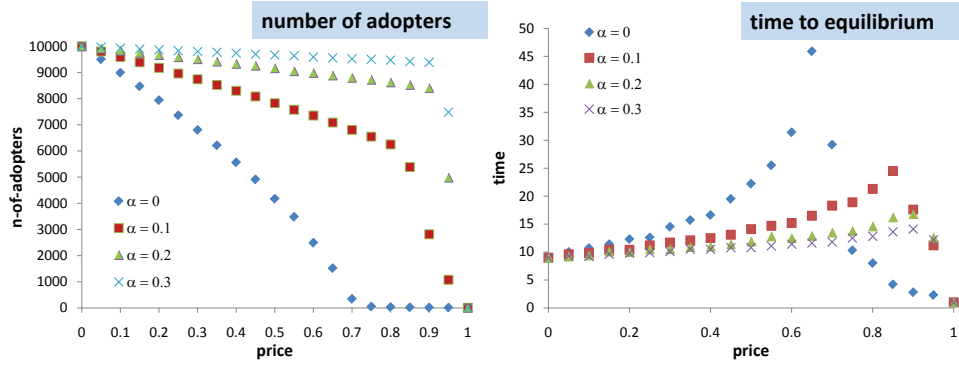


Figure 23: Percolation model with an endogenous learning curve. Poisson network with average degree 4. Left: final number of adopters. Right: diffusion time-length.

Fig. 22), this event has a larger impact than in other structures, such as the Poisson network (Fig. 23). The Poisson network is a fairly homogenous structure, and the link switching-on process of learning has a lower impact. This is particularly true at high learning rates. In a small world the marginal impact of increasing the learning rate is always large, while, in a Poisson random network the biggest improvement realizes already for $\alpha = 0.1$, while more sustained learning have an ever decreasing impact.

The picture is quite different for scale-free networks (Fig. 24). Here we notice the

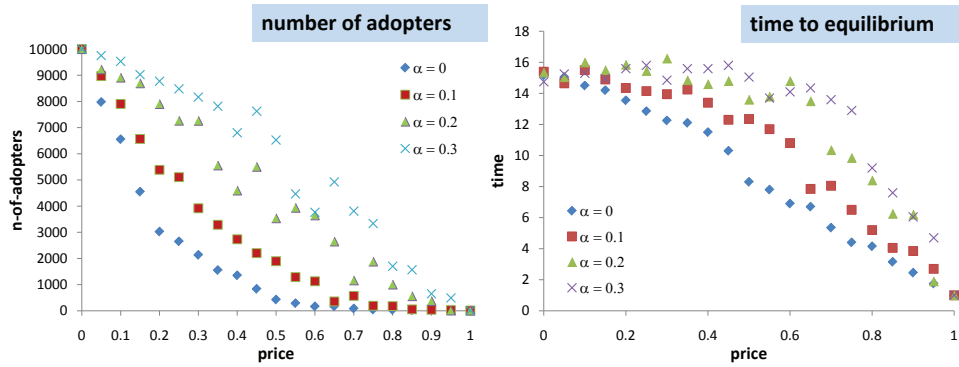


Figure 24: Percolation model with an endogenous learning curve. Scale-free network with average degree 2. Left: final number of adopters. Right: diffusion time-length.

following. First, learning has relatively little effect on scale-free networks compared to other structures. Second, the percolation threshold is less evident with learning, and almost no phase transition occurs above $\alpha = 0.2$. In other words, learning reduces the demand elasticity in a quite homogenous manner, so as to make the resulting demand curve quite linear. Third, in this scale-free network we find again a tight correspondence between extent and time of diffusion, as it is the case with regular one-dimensional lattices. In this case the explanation of a linear diffusion process is different: the reason is the very nature of a scale-free network, which is the lack of a characteristic scale. Learning enlarges the network by switching-on nodes (and links). This enlargement process is linear, and there is not a point where the increase in network size is higher or lower. We may say that percolation is a way to elicit the fundamental property of a scale-free network. A final remark is that simulation results have a large variability for scale-free networks. This is due to the randomness of the construction of a scale-free network, which is based on a preferential attachment stochastic process.

The conclusive message for this section is that learning curves favour innovation diffusion in very different ways depending on the network structure. In all cases though, learning curves are an adaptive price discrimination tool. In the case of a monopoly market for the innovation, the best strategy is to target consumers in connected network components, and initially charge a high price. Then let the price fall, and reach consumers with lower reservation prices. This allows to extract the most surplus from demand. This message is opposite to the popular idea based on which the innovator should initially price low, in order to launch the product and only later increase the price. Both strategies may be right, depending on the adoption mechanism at work. If there are strong network externalities that generate increasing returns on adoption (Katz and Shapiro, 1986; Arthur, 1989), the popular strategy makes sense. This is the case with communication technologies, for instance, as the fax machine or mobile telephones. Network externalities can be accounted for in a percolation model by having reservation prices go up with the number of adopters. Percolation results would be identical to what has been presented for learning curves, because the adoption decision is still based on comparing the product price with a reservation price: with learning curves the product price reaches reservation prices from above, while network externalities take reservation prices up to the product price. But the message for pricing is opposite in the two cases.

5 Conclusions

We have developed an agent-based percolation model to study how market network structure affects innovation diffusion. This study is relevant both for market diffusion efficiency and for the analysis of market demand. The variable of interests are the diffusion size (number of adopters) and the diffusion time-length. Our methodology is based on batch simulations, with results that are averages over a number of different simulation runs.

A percolation model combines two important factors of economic diffusion, which are information diffusion and individual preferences. This allows to obtain insightful results in an economic perspective. First of all, the percolation model is able to reproduce S-shaped time patterns of innovation adoption with a bottom-up approach. This result indicates that social network interactions are another factor shaping innovation diffusion dynamics beside rational expectations and learning mechanisms considered in the literature. Moreover, the percolation process shows a phase transition from a diffusion to a no-diffusion regime (phase), with a price threshold value which depends on the network structure. The percolation critical transition has two economic implications. First, it is a result of market inefficiency: a sizeable portion of the demand is not satisfied in the no-diffusion phase regime. Second, the demand curve is more elastic in the no-diffusion phase, and the market equilibrium price is lower.

The comparative analysis of percolation in different network structures indicates the main structural factors that affect innovation. We consider one-dimensional and two-dimensional regular lattices, Small-worlds networks, Poisson networks and Scale-free networks. The main results are the following. The higher average degree favours diffusion, as expected, but it is not the main driver. Structural factors may be more important. The crucial factor for diffusion in a percolation model is not the average path-length, but “connectivity spreading”, that is a topology where neighbours of successive order are increasing in number. Connectivity spreading is a diffusion multiplier, whose effect is striking when comparing a two-dimensional lattice to a one-dimensional lattice with the same degree. Spreading is also a characteristic of Poisson random network, but not of Small-worlds, where a relatively low average path-length is not enough to ensure high diffusion efficiency. In particular clustering is useless for diffusion in a percolation model: the triads that characterize clusters structures (where often two neighbours are also neighbour of each other) have redundant links, which do not contribute to diffusion efficiency. The reason is that in a percolation model links carry information, but adoption decisions are based only on individual preferences. This result is relevant to the debate regarding

whether clustering is good or bad for diffusion. In this perspective, a percolation model constitutes a clear benchmark: clustering is useless whenever adoption is based on individual preferences and not on social pressure. The final message from this analysis is that more individualist societies with high dimensionality are better markets for innovation diffusion.

One-dimensional lattices and scale-free networks surprisingly present one similar feature, a linear correspondence between diffusion size and time-length: whenever the diffusion extent is larger, also the time required to reach an equilibrium is longer, and viceversa. This is true also with learning. The explanation for this result is different for the two network structures. In a one-dimensional lattice connectivity does not “spread”, and adoption has only one route to travel. For scale-free networks the reason behind such linear correspondence between time and size of diffusion is the lack of a characteristic network scale, the main property of a scale-free network. These results indicate that although one-dimensional lattices and scale-free networks are very different structures, they can be two paradigmatic benchmarks for less-developed societies, with low dimensionality, high clustering, and a hierarchical structure. On the contrary non-linear diffusion patterns may be an indication of an individualist society, with low clustering and high dimensionality, better represented by a two-dimensional lattice or a Poisson network. Dimensionality is not captured by small-world networks, which are a popular model for many social systems. Beside connectivity spreading, dimensionality is the second main factor for innovation diffusion that we identify with our percolation model.

We have studied the effect of endogenous learning on percolation. During diffusion the learning feedback enlarges the network connected component of consumers willing to buy, and the percolation threshold shifts to a higher (initial) price. This has strong implications for market demand and pricing. The price elasticity of demand decreases, leading to a higher equilibrium price. In a monopoly market this effect can be used as a price discrimination tool: the best strategy is to start charging a high price, targeting consumers with high reservation price, and allow for a learning curve which adaptively reaches consumers with lower reservation price. This strategy allows to extract more surplus from the demand. For this strategy it is important that one consumer may be reached by the information about the new product more than one time. This is why learning has a strong impact on two-dimensional lattices and Poisson networks, and a relatively low impact on one-dimensional lattices and scale free networks. The effect of learning is particularly strong on small-world networks. Learning “switches-on” nodes

and consequently activates links progressively, with a strong effect on small-worlds where only few links are responsible of a relatively low average path-length. Again, clustering is unimportant for percolation also when learning is at work. Interestingly, learning has the lower effect in structures where there is a linear correspondence between diffusion size and diffusion time.

The two following tables summarize our findings by classifying the different network structures on four dimensions related to the percolation process. Table 1 reports on the

	<i>small diffusion size</i>	<i>large diffusion size</i>
<i>linear size-time relation</i>	One-dimensional lattices	Scale-free networks
<i>non-linear size-time relation</i>	Small-world networks	Two-dimensional lattices Poisson networks

Table 1: Classification of network structures based on the diffusion size and the relationship between the size and the time-length of diffusion.

diffusion size together with the relationship between diffusion size and diffusion time-length. Table 2 considers the impact of learning, and the effect of the number of seeds (initial adopters).

	<i>low impact of seeds</i>	<i>large impact of seeds</i>
<i>low impact of learning</i>	Scale-free networks	One-dimensional lattices
<i>large impact of learning</i>	Poisson networks Two-dimensional lattices	Small-world networks

Table 2: Classification of network structures based on the impact of endogenous learning and the impact of the number of initial adopters (seeds).

We conclude with some ideas for future research. The percolation model can accommodate the diffusion of multiple innovations. Goyal and Kearns (2012) study the strategic implications of diffusion for two competing innovative firms. An interesting question is whether competition affects diffusion, and how the percolation critical transition affects competition. Another possibility is to introduce local reinforcement beside the global feedback of learning curves. Local reinforcement can work at reservation prices level, accounting for imitation, peer-effect, and social pressure in general. A further modification involves the distribution of reservation prices. The uniform distribution defines a linear demand as the perfect information benchmark. A Pareto distribution (power-

law) better describes the social income distribution, and can shed light on the effects of economic inequality on innovation diffusion. The analytical study of percolation with non-uniform reservation price distribution can be approached with generating functions (Callaway et al., 2000). Finally, a big step for a theoretical model of innovation diffusion is to match empirical evidence. Percolation constitutes a simple benchmark of diffusion dynamics, with clear results regarding the effects of structural factors of social networks. A useful study would be to test innovation diffusion in a controlled web experiment (Centola, 2010), to see if the percolation mechanism is grounded on the evidence of human behaviour.⁷

References

- Abernathy, W.J., K. Wayne. 1974. Limits of the learning curve. *Harvard Business Review* **52**(5) 109–119.
- Albert, R., A.-L. Barabasi. 2002. Statistical mechanics of complex networks. *Reviews of Modern Physics* **74** 47–97.
- Argote, L., D. Epple. 1990. Learning curves in manufacturing. *Science* **247** 920–924.
- Arthur, B. 1989. Competing technologies, increasing returns, and lock-in by historical events. *Economic Journal* **99** 116–131.
- Banerjee, A., A. Chandrasekhar, E. Duflo, M. Jackson. 2012. The diffusion of microfinance. Tech. Rep. 17743, NBER.
- Barabasi, A.-L., R. Albert. 1999. Emergence of scaling in random networks. *Science* **286** 509–512.
- Bass, F. M. 1969. A new product growth for model consumer durables. *Management Science* **15**(5) 215–227.
- Blume, L. E. 1995. The statistical mechanics of best-response strategy revision. *Games and Economic Behavior* **11**(2) 111–145.
- Brock, W.A., S.N. Durlauf. 2010. Adoption curves and social interactions. *Journal of the European Economic Association* **8**(1) 232–251.

⁷The programmable simulation framework used for this article (*NetLogo*) is equipped with a technology for participatory model simulations on networked computers (*HubNet*).

- Callaway, D.S., M.E.J. Newman, S.H. Strogatz, D.J. Watts. 2000. Network robustness and fragility: percolation on random graphs. *Physical Review Letters* **85** 5468–5471.
- Campbell, A. 2012. Word-of-mouth communication and percolation in social networks. Tech. rep.
- Canepa, A., P. Stoneman. 2004. Comparative international diffusion: patterns, determinants and policies. *Economics of Innovation and New Technology* **13**(3) 279–298.
- Cantono, S., G. Silverberg. 2009. A percolation model of eco-innovation diffusion: the relationship between diffusion, learning economies and subsidies. *Technological Forecasting and Social Change* **76** 487–496.
- Centola, D. 2010. The spread of behavior in an online social network experiment. *Science* **329** 1194–1197.
- Choi, H., S.-H. Kim, J. Lee. 2010. Role of network structure and network effects in diffusion of innovations. *Industrial Marketing Management* **39** 170–177.
- Davies, S.W. 1979. Inter-firm diffusion of process innovations. *European Economic Review* **12**(4) 299–317.
- Davis, S., P. Trapman, H. Leirs, M. Begon, J.A. Heesterbeek. 2008. The abundance threshold for plague as a critical percolation phenomenon. *Nature* **454** 634–637.
- Erdos, P., A. Renyi. 1960. On the evolution of random graphs. *Publications of the Mathematical Institute of the Hungarian Academy of Sciences* **5** 17–61.
- Fogli, A., L. Veldkamp. 2012. Germs, social networks and growth. Tech. Rep. 9188, CEPR.
- Frenken, K., G. Silverberg, M. Valente. 2008. A percolation model of the product life-cycle. Tech. Rep. 73, UNU-MERIT.
- Geroski, P.A. 2000. Models of technology diffusion. *Research Policy* **29** 603–625.
- Goyal, S. 2007. *Connections: an introduction to the economics of networks*. Princeton University Press, Princeton, NJ.
- Goyal, S., M. Kearns. 2012. Competitive contagion in networks. Tech. rep.

- Griliches, Z. 1957. Hybrid Corn: An Exploration in the Economics of Technological Change. *Econometrica* .
- Hohnisch, M., S. Pittnauer, D. Stauffer. 2008. A percolation-based model explaining delayed takeoff in new-product diffusion. *Industrial and Corporate Change* **17**(5) 1001–1017.
- Iribarren, J.-L., E. Moro. 2011. *Affinity paths* and information diffusion in social networks. *Social Networks* **33** 134–142.
- Katz, M.L., C. Shapiro. 1986. Technology adoption in the presence of network externalities. *Journal of Political Economy* **94**(4) 822–841.
- Lee, E., J. Lee, J. Lee. 2006. Reconsideration of the winner-take-all hypothesis: complex networks and local bias. *Management Science* **52**(12) 1838–1848.
- Lopez-Pintado, D. 2008. Diffusion in complex social networks. *Games and Economic Behavior* **62** 573–590.
- Mansfield, E. 1961. Technical change and the rate of imitation. *Econometrica* **29**(4) 741–766.
- Moore, C., M.E.J. Newman. 2000. Epidemics and percolation in small-world networks. *Physical Review E* **61** 5678–5682.
- Morris, S. 2000. Contagion. *Review of Economic Studies* **67** 57–78.
- Newman, M.E.J., I. Jensen, R.M. Ziff. 2002. Percolation and epidemics in a two-dimensional small-world. *Physical Review E* **65**(2) 1–7.
- Pastor-Satorras, R., A. Vespignani. 2001. Epidemic spreading in scale-free networks. *Physical Review Letters* **86** 3200–3203.
- Pegoretti, G., F. Rentocchini, G. Vittucci-Marzetti. 2012. An agent-based model of innovation diffusion: network structure and coexistence under different information regimes. *Journal of Economic Interaction and Coordination* **7**(2) 145–165.
- Rogers, E.M. 1995. *Diffusion of innovations*. Free Press, New York.
- Silverberg, G., B. Verspagen. 2005. A percolation model of innovation in complex technology spaces. *J. of Economic Dynamics and Control* **29** 225–244.

- Solomon, S., G. Weisbuch, L. de Arcangelis, N. Jan, D. Stauffer. 2000. Social Percolation Models. *Physica A* **277** 239–247.
- Stauffer, D., A. Aharony. 1994. *Introduction to Percolation Theory*. Taylor & Francis, London.
- Stoneman, P. 2002. *The economics of technological diffusion*. Basil Blackwell, Oxford.
- Tirole, J. 1988. *The theory of industrial organization*. MIT Press, Cambridge, MA.
- Vega-Redondo, F., ed. 2007. *Complex Social Networks*. Cambridge University Press, Cambridge, England.
- Wasserman, S., K. Faust, eds. 1994. *Social Network Analysis*. Cambridge University Press, Cambridge, England.
- Watts, D. 2002. A simple model of global cascades on random networks. *Proceedings of the National Academy of Sciences* **99**(9) 5766–5771.
- Watts, D. J., S. H. Strogatz. 1998. Collective dynamics of ‘small-world’ networks. *Nature* **393** 440–442.
- Wright, T.P. 1936. Factors affecting the cost of airplanes. *Journal of Aeronautical Sciences* **3** 122–128.
- Young, H.P. 2009. Innovation diffusion in heterogenous populations: contagion, social influence and social learning. *American Economic Review* **99**(5) 1899–1924.